Mark Scheme (Results)

## Summer 2018

Pearson Edexcel GCE
In Mechanics M2 (6678/01)

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.


## PEARSON EDEXCEL GCE MATHEMATICS

## General Instructions for Marking

1. The total number of marks for the paper is 75 .
2. The Edexcel Mathematics mark schemes use the following types of marks:
'M' marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.
To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a 'force $x$ distance' term or 'mass x distance', if we allow them to cancel ' $g$ ' s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the $M$ mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an $M$ mark for solving the equations to find a particular quantity - this M mark is often dependent on the two previous M marks having been earned.

## ' A ' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.
'B' marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)
$A$ few of the $A$ and $B$ marks may be f.t. - follow through - marks.

## 3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes

- bod - benefit of doubt
- ft - follow through
- the symbol $\sqrt{ }$ will be used for correct ft
- cao - correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw - ignore subsequent working
- awrt - answers which round to
- SC: special case
- oe - or equivalent (and appropriate)
- dep - dependent
- indep - independent
- dp decimal places
- sf significant figures
-     * The answer is printed on the paper
- $\square$ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
6. If a candidate makes more than one attempt at any question:

- If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
- If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

## General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or $\sin$ ) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of $g=9.8$ should be given to 2 or 3 SF .
- Use of $\mathrm{g}=9.81$ should be penalised once per (complete) question.
N.B. Over-accuracy or under-accuracy of correct answers should only be penalised once per complete question. However, premature approximation should be penalised every time it occurs.
- Marks must be entered in the same order as they appear on the mark scheme.
- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),......then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads - if a misread does not alter the character of a question or materially simplify it, deduct two from any $A$ or $B$ marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations
$M(A)$ Taking moments about A.
N2L Newton's Second Law (Equation of Motion)
NEL Newton's Experimental Law (Newton's Law of Impact)
HL Hooke's Law
SHM Simple harmonic motion
PCLM Principle of conservation of linear momentum
RHS, LHS Right hand side, left hand side

| Q | Scheme | Marks | Notes |
| :---: | :---: | :---: | :---: |
| 1a |  |  |  |
|  | Motion down the plane: | M1 | Dimensionally correct. Condone sign errors and sin/cos confusion. |
|  | $F+750 \mathrm{~g} \sin \theta=1200$ | A1 | $(F+450=1200)$ |
|  | Use of $P=F v: F=\frac{9000}{v}$ | B1 | Award in (b) if not seen in (a) |
|  | $\frac{9000}{v}+750 g \times \frac{3}{49}=1200$ |  |  |
|  | $v=\frac{9000}{750}=12$ | A1 |  |
|  |  | (4) |  |
|  |  |  |  |
| 1b |  |  |  |
|  | $F=m a: F-(750 g \sin \theta+1200)=750 a$ | M1 | Dimensionally correct. Condone sign errors and $\sin /$ cos confusion. |
|  | $\frac{9000}{4.5}-\left(750 g \times \frac{3}{49}+1200\right)=750 a$ | A1 | Unsimplified equation with at most one error |
|  |  | A1 | Correct unsimplified equation |
|  | $a=0.47(0.467)\left(\mathrm{m} \mathrm{s}^{-2}\right)$ | A1 | 2 or 3 sf only not $\frac{7}{15}$ |
|  |  | (4) |  |
|  |  | [8] |  |


| Q | Scheme | Marks | Notes |
| :---: | :---: | :---: | :---: |
| 2 a | Using column vectors or $\mathbf{i}$ and $\mathbf{j}$ : $\binom{I \cos \theta}{I \sin \theta}=0.2\binom{7 \cos 35}{7 \sin 35}-0.2\binom{4}{0}$ | M1 | Must be subtracting. <br> Need both components <br> Could consider components separately |
|  | $\left(=\binom{0.347}{0.803}\right)$ | A1 | Correct unsimplified equation. Accept +/- |
|  | $\|I\|=\sqrt{0.347^{2}+0.803^{2}}$ | DM1 | Use Pythagoras to find magnitude Dependent on the previous M1 |
|  | $\|I\|=0.875$ | A1 | 0.87 or better |
|  |  | (4) |  |
| $\begin{gathered} \mathbf{a} \\ \text { alt } \end{gathered}$ | Alternative using vector triangle | M1 | Allow with velocities rather than impulse/momentum |
|  | Cosine rule: $\|I\|^{2}=1.4^{2}+0.8^{2}-2 \times 1.4 \times 0.8 \times \cos 35^{\circ}$ | $\begin{aligned} & \text { DM1 } \\ & \text { A1 } \end{aligned}$ | Dependent on the previous M1 |
|  | $\|I\|=0.875(\mathrm{~N} \mathrm{~s})(0.87$ or better $)$ | A1 |  |
|  |  | (4) |  |
| 2b | $\tan \theta=\frac{0.803}{0.347} \quad\left(\right.$ or $\left.\cos \theta=\frac{0.347}{0.875}\right)$ | M1 | Trig ratio of a relevant angle (using velocities or impulse/momentum) |
|  |  | A1ft | Correct expression for correct $\theta$ Ft on values from (a) Do not ISW |
|  | $\theta=66.6^{\circ}$ (67) | A1 | Or better from correct work. |
|  |  | (3) |  |
| $\begin{gathered} \mathbf{b} \\ \text { alt } \end{gathered}$ | Sine rule: $\frac{\sin \theta}{1.4}=\frac{\sin 35}{\|I\|}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1ft } \end{aligned}$ |  |
|  | $\theta=66.6^{\circ}$ (67) | A1 | Or better |
|  |  | (3) |  |
|  |  | [7] |  |


| Q | Scheme | Marks | Notes |
| :--- | :--- | :--- | :--- |
| 3a | Mass ratios : $4 a^{2}: \frac{\pi a^{2}}{2}: a^{2}\left(4+\frac{\pi}{2}\right)$ | B1 | Or equivalent |
|  | Distances relative to $B D: a,-\frac{4 a}{3 \pi},(d)$ | B1 | Or equivalent. Condone sign errors |
|  | Moments about $B D$ (or a parallel axis) | M1 | Dimensionally correct. All terms required. <br> Condone sign errors. Accept in a vector <br> equation. |
|  | $4 a^{2} \times a+\frac{\pi a^{2}}{2} \times \frac{-4 a}{3 \pi}=\left(4+\frac{\pi}{2}\right) a^{2} \times d$ | A1 | Correct unsimplified equation |


| Q | Scheme | Marks | Notes |
| :---: | :---: | :---: | :---: |
| 4 |  |  |  |
|  | $\begin{aligned} & \mathrm{M}(A): 2 a T=m g a \cos \theta \quad\left(T=\frac{1}{2} m g \cos \theta\right) \\ & \mathrm{M}(B): m g a \cos \theta+F r \times 2 a \sin \theta=R \times 2 a \cos \theta \end{aligned}$ | M1A1 | First equation <br> Need all terms. Condone sign errors and $\sin /$ cos confusion |
|  | Resolve $\leftrightarrow: \operatorname{Fr}=T \sin \theta\left(=\frac{1}{2} m g \cos \theta \sin \theta\right)$ | M1A1 | Second equation <br> Need all terms. Condone sign errors and sin/cos confusion |
|  | $\downarrow: R+T \cos \theta=m g$ | M1A1 | Third equation Need all terms. Condone sign errors and sin/cos confusion |
|  | Use $F r=\mu R: ~ \mu R=T \sin \theta$ | B1 | Condone correct inequality |
|  | Form equation in $\mu$ and $\theta$ : $\begin{aligned} & R=m g-\frac{1}{2} m g \cos \theta \cos \theta \\ & \quad \text { and } \quad \mu R=\frac{1}{2} m g \cos \theta \sin \theta \Rightarrow \end{aligned}$ | DM1 | Eliminate T and R <br> Dependent on first 3 M marks |
|  | $\mu=\frac{\frac{1}{2} m g \cos \theta \sin \theta}{m g-\frac{1}{2} m g \cos \theta \cos \theta}$ | DM1 | Solve for $\mu$ Dependent on previous M |
|  | $\mu=\frac{\cos \theta \sin \theta}{2-\cos ^{2} \theta}$ | A1 | Obtain given answer from correct working Must explain if inequality becomes equality |
|  |  | [10] |  |


| Alt <br> $\mathbf{1}$ | Moments (about $B$ ): <br> $m g a \cos \theta+F r \times 2 a \sin \theta=R \times 2 a \cos \theta$ | M 1 |  |
| :--- | :--- | :--- | :--- |
|  | Resolving (parallel to rod): <br> $F r \cos \theta+R \sin \theta=m g \sin \theta$ | A 1 | Correct unsimplified |
|  | Use of $F r=\mu R:$ <br> $m g \cos \theta+\mu R \times 2 \sin \theta=R \times 2 \cos \theta$ <br> $\mu R \cos \theta+R \sin \theta=m g \sin \theta$ | M 2 |  |
|  | Form equation in $\mu$ and $\theta:$ <br> $\frac{m g \sin \theta}{m g \cos \theta}=\frac{\mu R \cos \theta+R \sin \theta}{2 R \cos \theta-2 \mu R \sin \theta}$ <br> $\sin \theta$ <br> $\cos \theta$$=\frac{\mu \cos \theta+\sin \theta}{2 \cos \theta-2 \mu \sin \theta}$ | -1 each error |  |
|  | Solve for $\mu:$ <br> $2 \cos \theta \sin \theta-2 \mu \sin ^{2} \theta=\mu \cos ^{2} \theta+\cos \theta \sin \theta$ | DM1 | DM1 |
|  | $\mu=\frac{\sin \theta \cos \theta}{\cos { }^{2} \theta+2 \sin ^{2} \theta}=\frac{\sin \theta \cos ^{2} \theta}{2-\cos { }^{2} \theta}$ | A1 | Obtain given answer from correct working |
|  | NB for alternatives using moments and <br> resolving: <br> e.g. Resolve $\leftrightarrow: F r=T \sin \theta$ <br> $M(\operatorname{centre):~} a T=a \cos \theta R-a \sin \theta F r$ | First equation M1A1 <br> Sufficient equations to solve M2A2 |  |


| $\begin{aligned} & \hline \text { Alt } \\ & \mathbf{2} \end{aligned}$ |  |  | 3 concurrent forces |
| :---: | :---: | :---: | :---: |
|  | $\tan (\theta+\alpha)=\frac{\tan \theta+\tan \alpha}{1-\tan \theta \tan \alpha}$ | M1A1 |  |
|  | $\tan \theta=\frac{a}{2 a \tan \alpha} \Rightarrow \tan \alpha=\frac{1}{2 \tan \theta}$ | M1 |  |
|  | $\begin{aligned} \tan (\theta+\alpha) & =\frac{\tan \theta+\frac{1}{2 \tan \theta}}{1-\tan \theta \times \frac{1}{2 \tan \theta}} \\ & =2\left(\frac{\sin \theta}{\cos \theta}+\frac{\cos \theta}{2 \sin \theta}\right) \end{aligned}$ | $\begin{aligned} & \text { M1A1 } \\ & \text { A1 } \end{aligned}$ |  |
|  | $\begin{aligned} & F=\mu R \Rightarrow \\ & \qquad \mu=\frac{1}{\tan (\theta+\alpha)} \end{aligned}$ | B1 <br> DM1 |  |
|  | $=\frac{1}{2}\left(\frac{2 \sin \theta \cos \theta}{2 \sin ^{2} \theta+\cos ^{2} \theta}\right)=\frac{\cos \theta \sin \theta}{2-\cos ^{2} \theta}$ | $\begin{aligned} & \text { DM1 } \\ & \text { A1 } \end{aligned}$ | Obtain given answer from correct working |
|  |  | (10) |  |


| Q | Scheme | Marks | Notes |
| :---: | :---: | :---: | :---: |
| 5a |  |  |  |
|  | $\begin{aligned} & \text { CLM: } 3 m \times 2 u-2 m \times u=3 m v+2 m w \\ & (4 u=3 v+2 w) \end{aligned}$ | M1A1 |  |
|  | Impact law: $w-v=\frac{1}{3}(2 u+u)=u$ | M1A1 |  |
|  | Solve for simultaneous equations for $w$ or $v$ : $\begin{array}{r} 3 w-3 v=3 u, \quad 2 w+3 v=4 u \\ 5 w=7 u, w=\frac{7}{5} u \end{array}$ | $\begin{aligned} & \text { DM1 } \\ & \text { A1 } \end{aligned}$ | Dependent on both previous M marks <br> Must see working - Given Answer |
|  | $v=\frac{2}{5} u$ | A1 | Or equivalent. Must be positive |
|  |  | (7) |  |
| 5b | Speed of B after collision with wall: $\frac{1}{2} \times \frac{7}{5} u=\frac{7}{10} u$ | B1 | Accept +/- |
|  | Total time for either particle | B1 |  |
|  | Equate the time travelled for each particle: | M1 |  |
|  | $\frac{x}{\frac{7}{5} u}+\frac{y}{\frac{7}{10} u}=\frac{x-y}{\frac{2}{5} u}$ | A1 | Correct unsimplified |
|  | $\frac{5 x}{7 u}+\frac{10 y}{7 u}=\frac{5 x}{2 u}-\frac{5 y}{2 u}, \quad 10 x+20 y=35 x-35 y$ | DM1 | Dependent on previous M1 |
|  | $55 y=25 x, \quad y=\frac{5}{11} x$ | A1 | Or equivalent. $0.45 x$ or better |
|  |  | (6) |  |


| $\begin{gathered} \text { Alt } \\ 1 \end{gathered}$ | Speed of B after collision with wall: $\frac{1}{2} \times \frac{7}{5} u=\frac{7}{10} u$ | B1 | Accept +/- |
| :---: | :---: | :---: | :---: |
|  | Time of travel for B: $\frac{x}{\frac{7}{5} u}+\frac{y}{\frac{7}{10} u}=\frac{5 x+10 y}{7 u}$ | B1 |  |
|  | Distance moved by A: | M1 | Correct method for distance |
|  | $=\frac{2}{5} u \times\left(\frac{5 x+10 y}{7 u}\right)=\frac{2 x+4 y}{7}$ | A1 | Correct unsimplified |
|  | $\frac{2 x+4 y}{7}+y=x, 2 x+4 y+7 y=7 x$ | DM1 | Dependent on previous M1. Form equation in $x$ and $y$ |
|  | $y=\frac{5}{11} x$ | A1 | Or equivalent. $0.45 x$ or better |
|  |  | (6) |  |
| $\begin{gathered} \text { Alt } \\ 2 \end{gathered}$ | Speed of B after collision with wall: $\frac{1}{2} \times \frac{7}{5} u=\left(\frac{7}{10} u\right)$ | B1 | Accept +/- |
|  | $x$ - distance moved by $A=x-\frac{2}{5} u \times \frac{5 x}{7 u}=\frac{5}{7} x$ | B1 | Distance apart when $B$ hits the wall |
|  | Gap closing at $\frac{7}{10} u+\frac{2}{5} u=\frac{11}{10} u$ |  |  |
|  | Time to collision: $\left(\frac{5}{7} x\right) \div\left(\frac{11}{10} u\right)=\frac{50 x}{77 u}$ | M1A1 | Use of $\frac{9 x}{7}$ for $\frac{5 x}{7}$ is M0 |
|  | Distance moved by $B$ : $y=\frac{7}{10} u \times \frac{50 x}{77 u}=\frac{5}{11} x$ | $\begin{aligned} & \text { DM1 } \\ & \text { A1 } \end{aligned}$ | Dependent on previous M1 Or equivalent. $0.45 x$ or better |
|  |  | (6) |  |
| Alt 3 | Speed of B after collision with wall: $\frac{1}{2} \times \frac{7}{5} u=\left(\frac{7}{10} u\right)$ | B1 | Accept +/- |
|  | $x-$ distance moved by $A=x-\frac{2}{5} u \times \frac{5 x}{7 u}=\frac{5}{7} x$ | B1 | Distance apart when $B$ hits the wall |
|  | Ratio of speeds 4:7 | M1A1 |  |
|  | Distance moved by $B: y=\frac{7}{11} \times \frac{5 x}{7}=\frac{5}{11} x$ | $\begin{aligned} & \text { DM1 } \\ & \text { A1 } \end{aligned}$ | Dependent on previous M1 Use of $\frac{9 x}{7}$ for $\frac{5 x}{7}$ is M0 Or equivalent. $0.45 x$ or better |
|  |  | (6) |  |
|  |  | [13] |  |


| Alt <br> $\mathbf{4}$ | Speed of B after collision with wall: <br> $\frac{1}{2} \times \frac{7}{5} u\left(=\frac{7}{10} u\right)$ | B1 | Accept $+/-$ |
| :--- | :--- | :--- | :--- |
|  | $x$-distance moved by $A=x-\frac{2}{5} u \times \frac{5 x}{7 u}=\frac{5}{7} x$ |  |  | B1 | Distance apart when $B$ hits the wall |
| :--- | | Equate times for each particle to cover the <br> residual distance. <br> $\frac{5}{2 u}\left(\frac{5 x}{7}-y\right)=\frac{10}{7 u} \times y, \frac{1}{2}\left(\frac{5 x}{7}-y\right)=\frac{11}{7} y$ |
| :--- |
| Distance moved by $B: y=\frac{5}{11} x$ |


| Q | Scheme | Marks | Notes |
| :---: | :---: | :---: | :---: |
| 6 a | Differentiate $\mathbf{v}$ : $\quad \mathbf{a}=(4-6 t) \mathbf{i}+(-8+2 t) \mathbf{j}$ | M1A1 | Anywhere in (a) |
|  | Use of $\mathbf{F}=m \mathbf{a}$ and substitute $t=3$ : $\mathbf{F}=0.5((4-6 \times 3) \mathbf{i}+(-8+2 \times 3) \mathbf{j})=-7 \mathbf{i}-\mathbf{j}$ | DM1 | Dependent on the first M1 |
|  | Use of Pythagoras' theorem: | DM1 | Dependent on the first M1 |
|  |  |  | NB Could use Pythagoras and then use $\begin{array}{r} \mathbf{F}=m \mathbf{a} \cdot \begin{array}{r} 1^{\text {st }} \mathrm{M} 1-1^{\text {st }} \text { step. } \\ 2^{\text {nd }} \mathrm{M} 1-2^{\text {nd }} \text { step } \end{array} \end{array}$ |
|  | $\|\mathbf{F}\|=\sqrt{49+1}=\sqrt{50}(=5 \sqrt{2}=7.07 \ldots)$ | A1 | 7.1 or better |
|  | For $\mathbf{v}, \mathbf{i}$ component $=\mathbf{j}$ component: $\left(4 t-3 t^{2}\right)=\left(-40-8 t+t^{2}\right)$ | M1 | With no incorrect equations in $t$ seen |
|  | Solve for $t$ : $\begin{aligned} & 4 t^{2}-12 t-40=0, \Rightarrow t^{2}-3 t-10=0 \\ & (t-5)(t+2)=0, t=5 \end{aligned}$ | DM1 <br> A1 | Dependent on the previous M, Must see method if solving an incorrect quadratic Only - could be implied by later rejection of -2 |
|  | $\mathbf{a}=(4-30) \mathbf{i}+(-8+10) \mathbf{j}=-26 \mathbf{i}+2 \mathbf{j}\left(\mathrm{~ms}^{-2}\right)$ | A1 | Only |
|  |  | (9) |  |
| 6b | Integrate $\mathbf{v}$ : $\mathbf{r}=\left(2 t^{2}-t^{3}(+p)\right) \mathbf{i}+\left(-40 t-4 t^{2}+\frac{1}{3} t^{3}(+q)\right) \mathbf{j}$ | $\begin{aligned} & \text { M1 } \\ & \text { A2 } \end{aligned}$ | -1 ee |
|  | $\mathbf{r}_{1}=\mathbf{i}-43 \frac{2}{3} \mathbf{j}, \mathbf{r}_{2}=-93 \frac{1}{3} \mathbf{j} \quad \overrightarrow{A B}=\mathbf{r}_{2}-\mathbf{r}_{1}$ | DM1 | $\left(\frac{131}{3}, \frac{280}{3}\right)$ <br> Use limits in a definite integral or to evaluate a constant of integration Dependent on the previous M1 |
|  | $\overrightarrow{A B}=-\mathbf{i}-49 \frac{2}{3} \mathbf{j}\left(=-\mathbf{i}-\frac{149}{3} \mathbf{j}\right)$ | A1 | 49.7 or better |
|  |  | (5) |  |
|  |  | [14] |  |


| Q | Scheme | Marks | Notes |
| :---: | :---: | :---: | :---: |
| 7a | Horizontal distance: $x=u \cos \alpha t$ | B1 | $\frac{1}{\sqrt{5}} u t$ |
|  | Vertical distance: $y=u \sin \alpha t-\frac{1}{2} g t^{2}$ | M1A1 | $\frac{2}{\sqrt{5}} u t-\frac{1}{2} g t^{2}$ <br> Condone sign errors and sin/cos confusion |
|  | $\begin{aligned} & y=u \sin \alpha \times \frac{x}{u \cos \alpha}-\frac{g}{2} \times\left(\frac{x}{u \cos \alpha}\right)^{2} \\ & =x \tan \alpha-\frac{g x^{2}}{2 u^{2}} \times \frac{1}{\cos ^{2} \alpha}=2 x-\frac{g x^{2}}{2 u^{2}} \times \frac{1}{1 / 5} \end{aligned}$ | DM1 | Substitute for $t$ and $\alpha$ <br> Dependent on previous M1 |
|  | $=2 x-\frac{5 g}{2 u^{2}} x^{2}$ | A1 | Obtain given answer from exact working |
|  |  | (5) |  |
| 7b | $x=36, y=0: 0=2-\frac{5 g}{2 u^{2}} \times 36$, | M1 | Use given equation or a complete method using suvat to find $u$. |
|  | $u^{2}=\frac{5 g \times 36}{4}, \quad u=21\left(\mathrm{~m} \mathrm{~s}^{-1}\right)$ | A1 | Accept $\sqrt{45 g}$ |
|  |  | (2) |  |
| 7c | Min speed $=u \cos \alpha$ | M1 | $\left(u \cos \alpha=21 \times \frac{1}{\sqrt{5}}=9.39\left(\mathrm{~m} \mathrm{~s}^{-1}\right)\right)$ <br> Consistent with their B1 in (a) |
|  | Minimum KE: $\begin{equation*} \frac{1}{2} \times 0.3 \times(u \cos \alpha)^{2}=\frac{0.3}{2}\left(\frac{21}{\sqrt{5}}\right)^{2}=13.2 \tag{13} \end{equation*}$ | $\begin{array}{\|l} \text { DM1 } \\ \text { A1 } \end{array}$ | Dependent on previous M1 |
|  |  | (3) |  |
| $\begin{gathered} 7 \mathrm{c} \\ \text { alt } \end{gathered}$ | Max ht when $\frac{\mathrm{d} y}{\mathrm{~d} x}=0, \quad x=\frac{2 u^{2}}{5 g}(=18)$ | M1 | Or from $\frac{1}{2} \times 36$ (symmetry) |
|  | Conservation of energy: $\frac{1}{2} m u^{2}-m g h=\frac{1}{2} m v^{2}$ | M1 |  |
|  | $\begin{aligned} = & \frac{1}{2} \times 0.3 \times 21^{2}-0.3 \times g \times \frac{2 \times 21^{2}}{5 g} \\ = & 13.2(\mathrm{~J}) \end{aligned}$ | A1 |  |
|  |  | (3) |  |


| Q | Scheme | Marks | Notes |
| :---: | :---: | :---: | :---: |
| 7d | Gradientof trajectory at $B=-\frac{1}{2}$ | B1 | $\text { Accept } \frac{-1}{\tan \alpha}$ |
|  | Differentiate and equate : $\frac{\mathrm{d} y}{\mathrm{~d} x}=2-\frac{5 g}{u^{2}} x=-\frac{1}{2}$ | M1A1 | (their $u$ ) |
|  | solve for $x$ : $\quad-\frac{1}{2}=2-\frac{5 g}{u^{2}} x, \frac{5}{2}=\frac{5 g x}{u^{2}}$ | DM1 | Dependent on previous M1 |
|  | $x=\frac{21^{2}}{2 g}=22.5(23)(\mathrm{m})$ | A1 |  |
|  |  | (5) |  |
| $\begin{gathered} \text { 7d } \\ \text { alt1 } \end{gathered}$ | Gradient of trajectory at $B=-\frac{1}{2}$ | B1 | $\text { Accept } \frac{-1}{\tan \alpha}$ |
|  | Use components of velocity: $-\frac{1}{2}=\frac{u \sin \alpha-g t}{u \cos \alpha}$ | M1 |  |
|  | $t=\frac{u \sin \alpha+\frac{1}{2} u \cos \alpha}{g}\left(=\frac{105}{2 g \sqrt{5}}\right)$ | A1 | $(t=2.40)$ |
|  | Horizontal distance: $u \cos \theta t=22.5$ (23) (m) | $\begin{aligned} & \hline \text { DM1 } \\ & \text { A1 } \end{aligned}$ | Dependent on previous M1 |
|  |  | (5) |  |
| $\begin{gathered} 7 d \\ \text { alt2 } \end{gathered}$ | Gradient of trajectory at $B=-\frac{1}{2}$ | B1 | Can be implied by downward velocity $\frac{21 \sqrt{5}}{2} \text { or } \frac{u \cos \alpha}{\tan \alpha}$ |
|  | $v_{y}=-\frac{1}{2} \times 21 \times \frac{1}{\sqrt{5}},-\frac{21}{2 \sqrt{5}}=21 \times \frac{2}{\sqrt{5}}-g t$ | M1 | Use suvat to find $t$ |
|  | $t=\frac{\frac{5}{2} \times \frac{21}{\sqrt{5}}}{\rho}(=2.39)$ | A1 |  |
|  | Horizontal distance: $u \cos \theta t=22.5$ (23) (m) | DM1 | Dependent on previous M1 |
|  |  | A1 |  |
|  |  | (5) |  |
| $\begin{gathered} \text { 7d } \\ \text { alt3 } \end{gathered}$ | $\binom{u \cos \alpha}{u \sin \alpha} \cdot\binom{u \cos \alpha}{u \sin \alpha-g t}=0$ | B1 | Scalar product $=0$ |
|  | $\begin{gathered} u^{2}\left(\cos ^{2} \alpha+\sin ^{2} \alpha\right)-u \sin \alpha g t=0 \\ \Rightarrow u=\sin \alpha \cdot g t \end{gathered}$ | M1A1 | Must have $-g t$ in second vector Solve for $t$ |
|  | Horizontal distance: $u \cos \alpha . t=u \cos \alpha \times \frac{u}{g \sin \alpha}=\frac{21^{2}}{2 g}=22.5$ | M1A1 |  |
|  |  | (5) |  |
|  |  | [15] |  |

